’基于深度神经网络的SDE参数估计方法“为主题的研究计划书大纲

Most real-world problems are unavoidability subjected to random fluctuations, leading to a variety of complex phenomena. Examples cover fields from social sciences and natural sciences, including aerodynamics, neurodynamic, epidemiology, and finance. Deterministic relationships alone sometimes are insufficient to characterize the complex dynamics induced by the random fluctuations. Therefore, stochastic differential equations (SDE), which has been favored due to their powerful ability to express random perturbations acting on a system and their great flexibility in capturing system dynamics, are introduced to tackle these problems.

A stochastic differential equation (SDE) is a type of differential equation that includes a random or stochastic term. An SDE is typically written in the form:

where X(t) is the state of the system at time t, drift coefficient f(x(t), t) and diffusion coefficient g(x(t), t) are functions that describe the deterministic and stochastic components of the system, respectively, and dW(t) is a random increment of a Wiener process, which represents the random noise or uncertainty in the system.

Once a SDE have been properly designed for a specific problem, the dynamic of the systems can be captured by the value of the SDE's parameter. Therefore, the inverse problem aiming at estimating SDE's parameter from observation of real-world phenomena is of key importance in predicting future trends and supporting scientific decision-making.

To obtain a closed-form estimation, some researchers assume that the SDE is linear and random noises on the system are Wiener processes. In such cases, typical approaches such as the MLE is available, and the estimation is straightforward. However, there is no guarantee that every system can be described by such linear SDE properly. If the deterministic pattern of the system itself is not linear, then modeling with a linear model and performing parameter estimation will lose its meaning. In addition, the type of noise can also impress the accuracy of modeling. Taking the migration model of marine organisms as an example, GPS data can only be observed when the target organisms are close to the sea surface, resulting in that the movement trajectory exhibits jumps. This phenomenon is difficult to be modeled by the SDE with Wiener noise, while Levy noise is more appropriate.

As a matter of fact, one will always be faced with the tradeoff between the flexibility of the model and the difficulty in parametric estimation. For a SDE flexible enough to capture a real-world problem, its closed-form estimation is generally not available. As a result, various numerical estimating method have been proposed, such as Euler-Maruyama method, Ozaki method and EM algorithm. Most of these existing numerical methods have their drawbacks. For example, Euler-Maruyama method and Ozaki's method have strict assumption on the sample structure and is available only for high-frequency data. These assumptions often contradict reality and limit the application of the methods.

Deep neural networks have received extensive attention from academia and industry in recent years due to their powerful feature extraction capabilities. This study aims to construct deep neural networks-based method that overcome the complexity caused by the interaction of noise and nonlinear properties, and propose a stable, accurate and flexible method of joint parameter estimation for SDE in a general form.