’基于深度神经网络的SDE参数估计方法“为主题的研究计划书大纲

## Introduction

Most real-world problems are unavoidability subjected to random fluctuations, leading to a variety of complex phenomena. Examples cover fields from social sciences and natural sciences, including aerodynamics, neurodynamic, epidemiology, and finance. Deterministic relationships alone sometimes are insufficient to characterize the complex dynamics induced by the random fluctuations. Therefore, stochastic differential equations (SDE), which has been favored due to their powerful ability to express random perturbations acting on a system and their great flexibility in capturing system dynamics, are introduced to tackle these problems.

A stochastic differential equation (SDE) is a type of differential equation that includes a random or stochastic term. An SDE is typically written in the form:

where X(t) is the state of the system at time t, drift coefficient f(x(t), t) and diffusion coefficient g(x(t), t) are functions that describe the deterministic and stochastic components of the system, respectively, and dW(t) is a random increment of a Wiener process, which represents the random noise or uncertainty in the system.

Once a SDE have been properly designed for a specific problem, the dynamic of the systems can be captured by the value of the SDE's parameter. Therefore, the inverse problem aiming at estimating SDE's parameter from observation of real-world phenomena is of key importance in predicting future trends and supporting scientific decision-making.

To obtain a closed-form estimation, some researchers assume that the SDE is linear and random noises on the system are Wiener processes. In such cases, typical approaches such as the MLE is available, and the estimation is straightforward. However, there is no guarantee that every system can be described by such linear SDE properly. If the deterministic pattern of the system itself is not linear, then modeling with a linear model and performing parameter estimation will lose its meaning. In addition, the type of noise can also impress the accuracy of modeling. Taking the migration model of marine organisms as an example, GPS data can only be observed when the target organisms are close to the sea surface, resulting in that the movement trajectory exhibits jumps. This phenomenon is difficult to be modeled by the SDE with Wiener noise, while Levy noise is more appropriate.

As a matter of fact, one will always be faced with the tradeoff between the flexibility of the model and the difficulty in parametric estimation. For a SDE flexible enough to capture a real-world problem, its closed-form estimation is generally not available. As a result, various numerical estimating method have been proposed, such as Euler-Maruyama method, Ozaki method and EM algorithm. Most of these existing numerical methods have their drawbacks. For example, Euler-Maruyama method and Ozaki's method have strict assumption on the sample structure and is available only for high-frequency data. These assumptions often contradict reality and limit the application of the methods.

Deep neural networks have received extensive attention from academia and industry in recent years due to their powerful feature extraction capabilities. This study aims to construct deep neural networks-based method that overcome the complexity caused by the interaction of noise and nonlinear properties, and propose a stable, accurate and flexible method of joint parameter estimation for Levy SDE with switching.

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Here is a linear or non-linear function parameterized by . is a continuous time Markov chain with finite state space , where is a positive integer. The generator is the corresponding matrix with elements characterized by

For . is the driven Levy noise, which is Levy Processes with parameter . As an expected result of this work, the parameter can be estimated properly from high or low frequency equispaced observations . Here each sample is a single path observed at discrete time points and denotes the total sample size.

## Methodology

#### Propose contribution and performance measurements

##### Contribution of each term

The target SDE’s parameters consists of several components, the drift coefficients, the noise intensity and the parameter of the Levy process. Each components act together in different ways on the overall dynamics of the system. So it is necessary to propose a metric that can measure the contribution of each parts and see their impact on the estimation accuracy respectively.

[] has proposed a metric the contribution of parameters to the total dynamic for the linear SDE. However, since the target SDE in this work involves switching, the measurement is not straightforward. alpha does not directly affect the rate at which the system's dynamics are affected by its scale size, but affects the overall dynamics by altering the pattern of the system.

Therefore, we plan to propose a novel metric to measure each terms power considering the unique influence mode of the underlying Markov process, which is supposed to help concluding the influence the dynamics composition have on the estimation accuracy.

##### Performance Measurement